

## **ANALOG SIGNAL PROCESSING**

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## Summary

Signals are processed for a variety of reasons, such as to remove unwanted noise, to correct distortion, to make them suitable for transmission or to extract certain meaningful information. The term analog signal processing describes a body of techniques that can be implemented to process analog (or real-world) signals. Furthermore, in today's digital world, analog signal processing plays a fundamental role. It converts real-world information such as voice, sound, pressure, or voltage into the 1s and 0s of the digital domain so that it can be processed by today's powerful digital signal processors (DSPs). The conversion from 1s and 0s back into the real-world signals requires again analog signal processing. Without it, digital circuits would not be able to process real-world signals. As a result, most electronic equipment comprises analog processing circuitry that acts as interface between the digital-world and the real-world.

## 1. Introduction

A signal is a detectable (measurable) physical quantity containing some kind of information that can be recorded, conveyed, displayed, or manipulated. Examples of signals include temperature, pressure, sound waves, such as speech and music, electromagnetic waves, such as radio signals, and biomedical signals, such as the electroencephalogram (EEG) and electrocardiogram (ECG). Most of the signals naturally occurring in the world are analog, i.e., they provide a continuous stream of information on the physical quantity they represent. In this chapter, analog signals are generally limited to electrical variables such as voltage or current. Signals are processed

for a variety of reasons, such as to remove unwanted noise, to correct distortion, to make them suitable for transmission or to extract certain meaningful information. In the case of a cellular phone, for example, a signal of only a few micro-volts is detected by the antenna. Its noise content can be quite high because of unwanted interferences, hence requiring filtering. The term analog signal processing describes a body of techniques that can be implemented to process analog (or real-world) signals. This includes the theory and application of filtering, coding, transmitting, estimating, detecting, analyzing and reproducing analog signals. In this chapter, the concepts of linear time invariant systems that are fundamental to the theory of analog signal processing are first introduced. Filtering concepts are then presented along with detailed design information on practical analog signal processing and hardware implementations.

## 2. Analog Signals and Systems

### 2.1. Analog Signals

An analog signal is a physical quantity that conveys information about some physical phenomenon in a continuous nature. Common physical quantities include voltage and current, whereas physical phenomena can be temperature, position, sound, vibration, light intensity and so on. For example, an analog signal can be a voltage that may vary in amplitude, frequency or phase (in continuous range) in response to changes in temperature.

An analog signal is represented by a continuous function of one or more continuous, independent variables. The most commonly used variable is time. Hence the term *continuous-time* is often used to describe analog signals. This chapter will deal with one-dimensional analog signals that are a function of time. Analog signal can be described by a mathematical expression or a graphical plot (see Figure 1).

### 2.2. Analog Systems

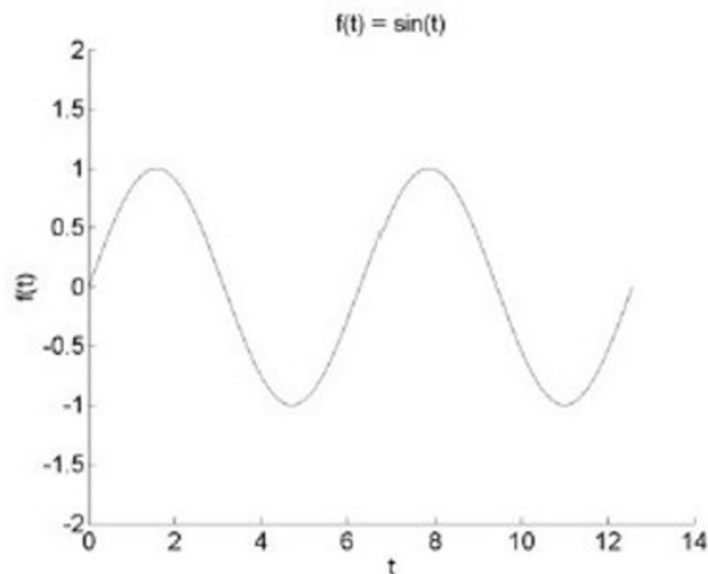


Figure 1: Example of an analog signal

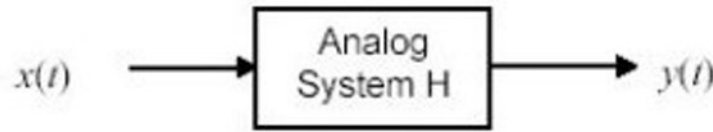


Figure 2: Block diagram of an analog system

An analog system is a collection and interconnection of several building blocks that performs some operation on an analog input signal to produce an analog output signal. The input signal is commonly known as the excitation signal, while the output or the processed signal is commonly referred to as the response signal. In the general case, a system may have multiple inputs as well as outputs.

Figure 2 depicts a block diagram representation of an analog system  $H$  that accepts a single analog excitation signal  $x(t)$  and then produces a single analog response signal  $y(t)$ .

The relationship between  $x(t)$  and  $y(t)$  is governed by the system  $H$  itself. Mathematically, it can be expressed as

$$y(t) = H[x(t)] \quad (1)$$

Analog systems can be classified as linear or non-linear, causal or non-causal, time-invariant or time-varying, stable or non-stable, and memoryless or with memory. Among them, systems that are linear and time invariant (or LTI) have special significance.

### 3. Linear Time Invariant Systems

A system is linear if it obeys the properties of homogeneity and additivity. Firstly, to show that a system  $H$  obeys the homogeneity property is to show that

$$H[\alpha x(t)] = \alpha H[x(t)] \quad (2)$$

for all scalar  $\alpha$  and input signal  $x(t)$ . Secondly, to show that a linear system  $H$  obeys the additivity property is to show that

$$H[x_1(t) + x_2(t)] = H[x_1(t)] + H[x_2(t)] \quad (3)$$

for all input signals  $x_1(t)$  and  $x_2(t)$ . These two properties, taken together, are known as the principle of superposition. The above two tests can be combined together so that a system  $H$  is said to be linear if and only if the following equation holds for all  $\alpha_1, \alpha_2,$

$x_1(t)$  and  $x_2(t)$  :

$$H[\alpha_1 x_1(t) + \alpha_2 x_2(t)] = \alpha_1 H[x_1(t)] + \alpha_2 H[x_2(t)] \quad (4)$$

Having linearity, complex input can be separated into simpler forms and the response of the system to this complex input can be calculated as the sum of the response to each of the simpler forms.

A system is time-invariant if the input-output relationship does not change with time. In other words, if  $y(t) = H[x(t)]$  then  $y(t-\tau) = H[x(t-\tau)]$  for all  $\tau$ .

A linear time-invariant (LTI) system is one which is both linear and time-invariant. This particular class of system forms the basis of several useful methods of system analysis and plays a major role in the design and analysis of communication, control and signal processing systems. Although most practical systems are non-linear, it is much easier to analyze them using an LTI approximation around their operating points.

Signals and systems can be represented either in time or frequency related transform domain. Some problems can be solved more easily in one than the other.

### 3.1. Time Domain Analysis

#### 3.1.1. Impulse Response

Any LTI system may be represented in the time domain by its response to a specific signal called the unit impulse, or Dirac delta function. The Dirac delta function, denoted by  $\delta(t)$ , is defined as

$$\int_a^b \delta(t) x(t) dt = x(0), \quad a < 0 < b \quad (5a)$$

provided  $x(t)$  is continuous at  $t = 0$ . The delta function is zero everywhere except at the origin where it is infinitesimally narrow while its amplitude is infinite (see Figure 3), and the total area under the impulse is unity, hence the name unit impulse; that is,

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (5b)$$

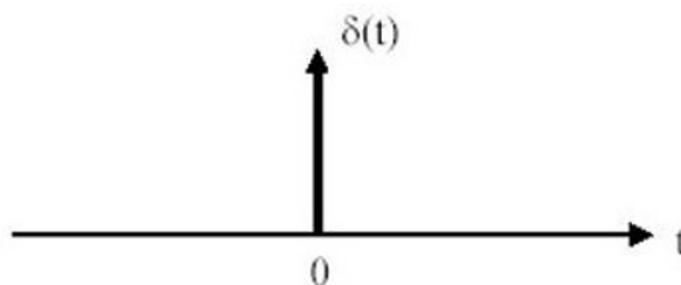


Figure 3: Unit impulse occurring at  $t=0$

Suppose  $h(t)$ , as shown in Figure 4, is the response of an analog LTI system  $H$  to a unit impulse  $\delta(t)$ .

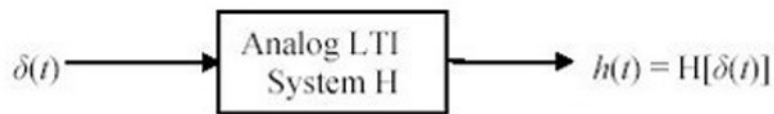


Figure 4: Impulse response

Then by knowing  $h(t)$ , the behavior of the system  $H$  with input signal  $x(t)$  and output signal  $y(t)$  can be described as

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \quad (6)$$

The right hand side of this equation, known as the **convolution integral**, shows that the impulse response  $h(t)$  is a complete characterization of the LTI system. Eq. (6) can also be written by using the convolution operator  $*$  as

$$y(t) = x(t) * h(t) \quad (7)$$

Note that in practical system design, the causality and stability of the system are required. A system is considered causal if and only if the current output does not depend on any future values of the input. For physically realizable LTI systems, a necessary and sufficient condition for causality is that its impulse response be zero prior to application of the excitation, that is

$$h(t) = 0 \text{ for } t < 0 \quad (8)$$

To be practically useful, a system must also be stable in the sense that a bounded input always leads to a bounded output. For LTI systems, a sufficient condition for this bounded-input, bounded-output (BIBO) stability is that the area under the magnitude of its impulse response be finite, that is  $h(t)$  must be absolutely integrable with

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty \quad (9)$$

### 3.1.2. Differential Equation Representation

Although LTI systems are completely characterized by their impulse response, in practice they are often described by linear constant-coefficient differential equations.

An analog LTI system with excitation signal  $x(t)$  and response signal  $y(t)$  can be modeled using a linear constant-coefficient ordinary differential equation (ODE):

$$\begin{aligned} \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_1 \frac{dy(t)}{dt} + a_0 y(t) \\ = b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \dots + b_0 x(t) \end{aligned} \quad (10)$$

where  $a_k$  and  $b_k$  are constant coefficients.

The response of the system can be obtained by solving the above equation.

### 3.2. Transform Domain Analysis

Solving differential equations or looking for impulse responses in the time domain is often a tedious task. However, this can be overcome by analyzing the system in the transform domain using Laplace or Fourier transform.

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#### Bibliography

Ambaradar, A. (1999), *Analog and Digital Signal processing*, Pacific Grove: Books/Cole Publishing Co.. [This book provides an excellent coverage of the fundamental principles and applications of both analog and digital signal processing]

Budak, A. (1974), *Passive and Active Network Analysis and Synthesis*, Houghton Mifflin. [This book covers the analysis and synthesis of passive as well as active networks]

Bult, K. and H. Wallinga, (1986) "A CMOS four-quadrant analog multiplier," *IEEE Journal of Solid-State Circuits*, vol. SC-21, no. 3, pp. 430-435. [This article presents a four-quadrant multiplier based on the square-law characteristics of the MOSFET transistor]

Chen, W.-K. (1995), *The Circuits and Filters Handbook*, CRC Press. [This is a very handy reference book that covers electrical circuits and filters]

Chen, W. K. (1986), *Passive and Active Filters*, John Wiley & Sons. [This book offers a good introduction to the theory and design of modern analog filters. Section 5.5.2 is in part adapted from this book.]

Franco, S. (1998), *Design with Operational Amplifier and Analog Integrated Circuits*, McGraw-Hill International. [This book provides a good review on operational amplifiers as well as analog integrated circuits]

Han, G. and E. Sanchez-Sinencio (1998), "CMOS Transconductance Multipliers: A tutorial", *IEEE Transactions on Circuits and Systems Part II*, vol. 45, no.12, pp.1550-1563. [This article is an excellent tutorial on analog multipliers]

Haykin, S. and B. Van Veen (2003), *Signals and Systems*, John Wiley and Sons. [This book offers good introductory treatment to transform domain analysis]

Jaeger, R. C. (1997), *Microelectronic Circuit Design*, McGraw-Hill. [This is a good textbook to develop a comprehensive understanding of the basic techniques of modern electronic design: analog and digital, discrete and integrated]

Johns, D. A. and K. Martin (1997), *Analog Integrated Circuit Design*, John Wiley & Sons. [This book provides a comprehensive overview of analog integrated circuits such as analog integrated filters. Section 5.6.2 is in part adapted from this book.]

Oppenheim, A. V., A. S. Willsky and S. H. Nawab (1997), *Signals and Systems*, Prentice-Hall. [This is a good reference book on signals and systems]

Phillips, C. L., Parr, J. M. and E. A. Riskin, *Signals, Systems and Transforms*, Prentice-Hall, 2003. [This book provides a good introduction to the topics of signals and systems]

Razavi B.(1998), *RF Microelectronics*, Prentice-Hall. [This book deals with the analysis and design of RF integrated circuits and systems]

Soliman, S. S. and H. D. Srinath (1998), *Continuous and Discrete Signals and Systems*, Prentice-Hall. [This books deals with signals and systems in both analog and digital means]

Tsividis, Y. P. and J. O. Voorman (1993), *Integrated Continuous-Time Filters: Principles, Design, and Applications*, IEEE Press. [This volume regroups key articles on successful VLSI integration of continuous-time filters]

Van Valkenburg, M. E. (1982), *Analog Filter Design*, HRW. [This book is an excellent reference for electrical engineers called upon designing filters]

### Biographical Sketches

**Bouzerdoun** is Professor of Computer Engineering and Head of School of Electrical, Computer and Telecommunications Engineering at the University of Wollongong. He received the Masters and Ph.D. degrees, both in electrical engineering, from the University of Washington, Seattle, in 1986 and 1991, respectively. In 1991 he joined the University of Adelaide as a Postdoctoral Fellow; then he was appointed Lecturer in 1992 and Senior Lecturer in 1995. In 1998 he was appointed Associate Professor at Edith Cowan University, and in 2004 he moved to University of Wollongong. He has received several distinguished awards; amongst them are the Vice Chancellor's Distinguished Researcher Award in 1998 and 1999, and the awards for Excellence in Research Leadership and Excellence in Postgraduate Supervision in 2000. In 2001 he was awarded a Distinguished Researcher Fellowship (Chercheur de Haut Niveau) from the French Ministry of Research to spend three months at a National Research Centre Laboratory LASS-CNRS in Toulouse. In 2004 he was a visiting professor at L2TI, Institut Galilée, Université Paris 13. Dr. Bouzerdoun is a senior member of IEEE, a member of INNS, and an Associate Editor for the IEEE Transactions on Systems, Man and Cybernetics. From 1993 to 1997 he was a member of the IEEE South Australian Section and served as Chair of the IEEE Neural Network Council Region Interest Group (RIG) from 1995 to 1997. He authored over 200 technical articles in journals, book chapters and conference proceedings. His research interests include machine vision and pattern recognition, neural networks and their applications to signal and image processing, and VLSI implementation of smart micro-sensors.

**Chai** completed the BE(Hons) and PhD degrees in Electrical & Electronic Engineering from The University of Western Australia (UWA), Australia, in 1994 and 1999, respectively. He is currently a Research Fellow with the School of Engineering and Mathematics at Edith Cowan University since 1999. He is also the Director of the Visual Information Processing Research Group. From 1998 to 1999, he was a Research Associate at UWA, where he engaged in MPEG-4 codec implementation project with the Cooperative Research Centre for Broadband Telecommunications and Networking. Dr. Chai is currently the WA Chapter Chair of the IEEE Signal Processing Society. As well, he is the Past Chair of the IEEE Western Australia Section (2005); was the Chair in 2003-2004, Vice-Chair in 2002 and Secretary in 2000-2002. He has published over 40 technical papers and has won two ARC Discovery Project research grants since 2003. His research interests include image and signal processing, video coding, and multimedia communications.

**Boussaid** received the engineering degree in electronics with honors from the Université des Sciences et



de la Technologie Houari Boumediène, Algeria, in 1995, and the M.S. and Ph.D. degrees in microelectronics in 1996 and 1999, respectively, both from the Institut National des Sciences Appliquées (INSA) of Toulouse. From May 1999 to February 2000, he was a Research Associate within the Microsystems and Microstructures research group at the French National Centre for Scientific Research (LAASCNRS), in France. In March 2000, he joined Edith Cowan University, as a Post-Doctoral Research Fellow and a member of the Visual Information Processing Research Group. In December 2001, he was the recipient of an Australian Research Council APD Fellowship to develop a new generation of smart vision sensors featuring on-chip and pixel-level implementation of human vision based algorithms. In January 2005, he joined the University of Western Australia as a Lecturer. His research interests include smart CMOS vision sensors, neuromorphic systems, device simulation, modeling and characterization in deep sub-100nm CMOS processes.